

ISSN 0354-9836

UDC: 621

THERMAL SCIENCE

1/2010

THERMAL SCIENCE VOL. 14 NUMBER 1 PP. 1-290 BELGRADE 2010

Editor-in-Chief Prof. Simeon Oka, Ph. D., Vinča Institute of Nuclear Sciences, Belgrade

National Editorial Board
Vukman Bakić, Ph. D., editor, Vinča Institute of Nuclear Sciences, Belgrade
Predrag Stefanović, Ph. D., Vinča Institute of Nuclear Sciences, Belgrade
Prof. Vladimir Stevanović, Ph. D., Faculty of Mechanical Engineering, Belgrade
Prof. Dragoslava Stojilković, Ph. D., Faculty of Mechanical Engineering, Belgrade
Mladen Ilić, Ph. D., Energy Saving Group, Belgrade
Dejan Cvetinović, M. Sc., Vinča Institute of Nuclear Sciences, Belgrade

Regional Editorial Board
Prof. Neven Duić, Ph. D., Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Zagreb, Croatia
Assoc. Prof. Iztok Golobič, Ph. D., Faculty of Mechanical Engineering, University of Ljubljana, Ljubljana, Slovenia
Prof. Željko Grbavčić, Ph. D., Faculty of Technology and Metallurgy, University of Belgrade, Belgrade, Serbia
Petar Gvero, Ph. D., Faculty of Mechanical Engineering, University of Banja Luka, Banja Luka, Republic of Srpska (B&H)
Prof. Jordan Hristov, Ph. D., Department of Chemical Engineering, Sofia University of Technology and Metallurgy, Sofia, Bulgaria
Prof. Emmanuel Kakaras, Ph. D., Department of Mechanical Engineering, National Technical University of Athens, Athens, Greece
Prof. Nenad Kažlć, Ph. D., Faculty of Mechanical Engineering, University of Podgorica, Podgorica, Montenegro
Valentino Stojkovski, Ph. D., Faculty of Mechanical Engineering, University "St. Kiril and Metodij", Skopje, Republic of Macedonia
Prof. Paul Vasilescu, Ph. D., Chemical Engineering Department, "Politehnica" University of Bucharest, Bucharest, Romania

International Advisory Board of the Journal THERMAL SCIENCE

Andersson, S., Department of Energy Conversion Chalmers University of Technology Göteborg, Sweden	Johanson, J. E., Department of Chemical Engineering Technical University of Denmark Lyngby, Denmark	Novak, P. D., School of Technologies and Systems Novo Mesto, Slovenia
Anthony, E. J., CETC, NRCAN Ottawa, Ont., Canada	Jovanović, J., Chair for Fluid Mechanics Friedrich-Alexander University Erlangen, Germany	Ots, A., Power Engineering Department Tallinn Technical University Tallinn, Estonia
Baskakov, A. P., Heat Power Department Ural Polytechnical Institute Ekaterinenburg, Russia	Kanevče, G., Faculty of Engineering University of Bitola Bitola, Republic of Macedonia	Pokusaev, B. G., Chair of Thermodynamics and Heat Transfer, Moscow Institute for Chemical Mechanical Engineering Moscow, Russia
van den Bleek, C. M., Department of Chemical Engineering Delft University of Technology Delft, The Netherlands	Leckner, B., Department of Energy Conversion Chalmers University of Technology Göteborg, Sweden	Radulović, P., Advanced Combustion Engineering Research Center Brigham Young University Provo, UT, USA
Borodulya, V. A., A. V. Luikov Heat and Mass Transfer Institute Minsk, Belarus	Martinenko, O., A. V. Luikov Heat and Mass Transfer Institute Minsk, Belarus	Renz, U., R-W. Chair for Heat Transfer and Climatization Technical University Aachen, Germany
Burdukov, A. P., Institute of Thermophysics Siberian Branch of the Russian Academy of Sciences Novosibirsk, Russia	Mayinger, F., Chair for Thermodynamics Technical University Munich Munich, Germany	Riznić, J., Operational Engineering Assessment Division, Canadian Nuclear Safety Commission Ottawa, Ont., Canada
Durst, F., Chair for Fluid Mechanics Friedrich-Alexander University Erlangen, Germany	Miccio, M., Department of Chemical and Food Engineering University of Salerno Salerno, Italy	Šašić, S., Department of Applied Mechanics Chalmers University of Technology Göteborg, Sweden
Hanjalić, K., Faculty of Applied Sciences, Delft University of Technology, Delft, The Netherlands	Mikić, B., Massachusetts Institute of Technology Cambridge, Mass., USA	Sekulić, D., Department of Mechanical Engineering University of Kentucky Lexington, Ken., USA
Horio, M., Department of Chemical Engineering Tokyo University of Agriculture and Technology Tokyo, Japan	Mujumdar, A. S., Department of Mechanical and Production Engineering National University of Singapore Singapore	Volchkov, E. P., Institute of Thermophysics Siberian Branch of the Russian Academy of Sciences Novosibirsk, Russia
Howard, J. R., Aston University Birmingham, UK	Nakoryakov, V. E., Institute of Thermophysics Siberian Branch of the Russian Academy of Sciences Novosibirsk, Russia	van Wachem, B., Imperial College London Department of Mechanical Engineering London, England
Ishii, M., School of Nuclear Engineering Purdue University W. Lafayette, Ind., USA		

THERMAL SCIENCE

ISSN 0354-9836 ❖ UDC 621 ❖ Vol. 14 ❖ No. 1 ❖ Belgrade 2010

Founder:

Society of Thermal Engineers of Serbia

Publisher:

Vinča Institute of Nuclear Sciences

The edition is supported by:

Ministry of Science and Technological Development of the Republic of Serbia, and the
Laboratory for Thermal Engineering and Energy of the
Vinča Institute of Nuclear Sciences

<http://thermalscience.vinca.rs>

CONTENTS

Jun Li, Lingen Chen, Fengrui Sun

Maximum work output of multistage continuous Carnot heat engine system with finite reservoirs of thermal capacity and radiation between heat source and working fluid 1-9

*Praveen K. Sharma, Ram P. Prajapati,
Rajendra K. Chhajlani*

Rayleigh-Taylor instability of two superposed magnetized viscous fluid with suspended dust particles 11-29

Sunday E. Etuk, Lous E. Akpabio, Ita O. Akpan

Comparative study of thermal transport in *Zea mays* straw and *Zea mays* heartwood (cork) boards 31-38

Raghavan Vijayan, Pss Srinivasan

Experimental evaluation of internal heat exchanger influence on R-22 window air conditioner retrofitted with R-407C 39-47

Khaled S. Shibib, Mohammed A. Munshid

Thermal behavior of tissues having different porosities during continuous CO₂ laser irradiation 49-56

Jahar Sarkar, Souvik Bhattacharyya, Mudali Ramgopal

Experimental investigation of transcritical CO₂ heat pump for simultaneous water cooling and heating 57-64

<i>Chandrasekar Murugesan, Suresh Sivan</i>	
Limits for thermal conductivity of nanofluids	65-71
<i>Rajamanickam Muthucumaraswamy, Kailasam Sathappan, Ramasamy Natarajan</i>	
Diffusion and heat transfer effects on exponentially accelerated vertical plate with variable temperature	73-77
<i>Said Agamy, Adul Mohsen Metwally, Amir Mohammad Al-Ramady, Sayed Mohamed Elaraby</i>	
A RELAP5 model for the thermal-hydraulic analysis of a typical pressurized water reactor	79-88
<i>Slobodan R. Savić, Branko R. Obrović Dušan R. Gordić, Saša B. Jovanović</i>	
<u>Investigation of the ionized gas flow adjacent to porous wall in the case when electroconductivity is a function of the longitudinal velocity gradient</u>	89-102
<i>Ahmed A. Kohil, Hassan A. Farag, Mona E. Ossman</i>	
Mathematical modeling of a multi-stream brazed aluminum plate fin heat exchanger	103-114
<i>Weijan Shen, Fock-Lai Tan</i>	
Thermal management of mobile devices	115-124
<i>Ameni Mokni, Hatem Mhiri, Georges Le Palec, Philippe Bournot</i>	
Turbulent mixed convection in heated vertical channel	125-135
<i>Shyam Sunder Tak, Rajeev Mathur Rohit Kumar Gehlot, Aiyub Khan</i>	
MHD free convection-radiation interaction along a vertical surface embedded in Darcian porous medium in presence of sores and Dufour's effects	137-145
<i>Haris Sivasankaran, Godson Asirvatham, Jefferson Bose, Bensely Albert</i>	
Experimental analysis of parallel plate and crosscut pin fin heat sinks for electronic cooling applications	147-156

<i>Ioan V. Luminosu, Coleta T. De Sabata Aldo I. De Sabata</i>	
Research in solar energy at the "Politehnica" University of Timisoara: Studies on solar radiation and solar collectors	157-169
<i>Almed Yousof Bakier</i>	
Effect of thermophoresis on natural convection boundary layer flow of a micropolar fluid.	171-181
<i>Afshin Mohsenzedli, Mousa Farhadi, Kurosh Sedighi</i>	
Convective cooling of tandem of heated triangular cylinders placed in a channel	183-197
<i>Žarko M. Stevanović, Nikola S. Mirkov, Žana Ž. Stevanović, Andrijana D. Stojanović</i>	
Validation of atmospheric boundary layer turbulence model by on-site measurements	199-207
<i>Thakur Pankaj</i>	
Elastic-plastic transition stresses in a thin rotating disc with rigid inclusion by infinitesimal deformation under steady-state temperature	209-219
<i>Rachid Saim, Said Abboudi, Boumediene Benyoucef</i>	
Computational analysis of transient turbulent flow and conjugate heat transfer characteristics in a solar collector panel with internal, rectangular fins and baffles	221-234
<i>Vukić N. Lazić, Aleksandar S. Sedmak, Miroslav M. Živković, Srbislav M. Aleksandrović, Ratko D. Čukić, Radomir D. Jovičić, Ivana B. Ivanović</i>	
Theoretical-experimental determining of cooling time (t_{8-5}) in hard facing of steels for forging dies	235-246
<i>Dušan S. Dinilović, Vesna D. Karović Maričić, Vojin B. Čokorilo</i>	
Solving paraffin deposition problem in tubing by heating cable application	247-253
<i>Elvir H. Zlomušica</i>	
Wind energy resources in Bosnia & Herzegovina	255-260
<i>Milić D. Erić, Dejan B. Cvetinović, Predrag Lj. Stefanović, Predrag M. Radovanović, Nikola V. Živković</i>	
Investigation of pressure pulsations in the furnace and flue gas tract of the pulverized coal combustion utility boiler.	261-270

*Peng Xu, Arun S. Mujumdar,
Hee Joo Poh, Boming Yu*

Heat transfer under a pulsed slot turbulent impinging
jet at large temperature differences 271-281

*Qiang Xu, Xinggul Que, Liying Cao,
Yong Jiang, Cong Jin*

Total heat flux on the wall: Bench scale wood crib fires tests 283-290

Comments and discussions

Fergal Hourigan

3-D CFD analysis on effect of hub-to-tip ratio on
performance on impulse turbine for wave energy conversion I-III

National Editorial Board address	Journal <i>Thermal Science</i> P. O. Box 522, 11001 Belgrade, Serbia Phone: (381-11) 245-56-63 • Fax: (381-11) 245-36-70 E-mail: okasn@rcub.bg.ac.rs
Editorial secretary	Ljiljana Šopalović, dipl. sociologist
Technical editor	Vladimir Živković
Web master	Zoran Mihailović
UDC	Radmila Lučić, M. Sc.
Computer work	Stanka Petrović

Printed by: Vinča Institute of Nuclear Sciences, 12-14, M. Petrovića-Alasa Str., Vinča, Belgrade
in 400 copies, April 2010 ❖ Phone: (381-11) 806-67-46

Subscription

Annual – four issues	110 €/174 \$
with air-mail delivery (Europe)	121 €/191 \$
with air-mail delivery (overseas)	129 €/204 \$
Single issue	30 €/47 \$
with air-mail delivery (Europe)	33 €/52 \$
with air-mail delivery (overseas)	35 €/55 \$

Subscriptions has to be made in written form on the E-mail address bakicv@vinca.rs or via the web site of the journal *Thermal Science* <http://thermalscience.vinca.rs>
For domestic subscribers the prices are 110 € (annual), or 30 € per issue, in domestic currency, according to the Euro/Din equivalence on the day of payment, on the account 205-39132-62, Društvo termičara Srbije, Beograd.

On written request we will send an invoice.

Advertisements (by written request on E-mail address bakicv@vinca.rs)

On the IV cover page (in black) over the standard color of the journal	400 €/632 \$
On the III cover page (in black)	350 €/553 \$
One page in the journal body (in black)	250 €/395 \$

For advertisements in other colors or in full color, special proforma invoice will be send after receiving the requested lay-out.

For repeated advertisement without changes – 30% price reduction.

CIP – Каталогизација у публикацији
Народна библиотека Србије, Београд

621.1

THERMAL Science : International Scientific
Journal / editor-in-chief Simeon Oka. - Vol. 1,
No. 1 (1997)-.-Belgrade : Vinča Institute of
Nuclear Sciences, 1997-. - 24 cm

Tromesečno
ISSN 0354-9836 = Thermal science
COBISS.SR-ID 150995207

Information and guidelines for authors

In the journal *Thermal Science* you can publish original scientific papers, previous communications, reviews and professional papers, as well as short reviews of new books and scientific meetings in the fields of fundamental thermal sciences (fluid mechanics, heat and mass transfer, combustion, thermodynamics, chemical processes), important for thermal and chemical engineering, process engineering, energy and power engineering and ecology, and related engineering fields. Following modern trends, besides classical problems of turbulent flow, heat or mass transfer, and combustion, the journal *Thermal Science* started to cover many new engineering topics: biomass combustion, energy efficiency and sustainable development, use of renewable energy sources, heat and mass transfer in fires, thermodynamic optimization of the processes using entropy generation minimization, and many others (fluid flow and heat transfer in nanotechnologies, heat transfer in electronic systems, flow, heat transfer and combustion in internal combustion engines, magneto hydrodynamics, heat transfer and processes in buildings, *etc.*).

Short abstracts of the M. Sc. and Ph. D. thesis defended at Universities from the countries of the Balkan Region, or done by the young researchers from Balkan Region at the Universities worldwide, are also welcome.

Manuscript will be considered only on the understanding that it is not currently being submitted to other journals.

Submission of an article implies that the results described has not been published previously (except in the form of an abstract or as part of a published lecture or academic thesis), that it is not under consideration for publication elsewhere, that its publication is approved by all authors and that, if accepted, it will not be published elsewhere in the same form, in English or in any other language, without the written consent of the Publisher.

All papers will be sent for peer reviews, to two independent experts. Authors are obliged to follow remarks and comments of the reviewers, as well as to follow Instructions for preparing manuscripts, and technical remarks and corrections of the Editorial Board.

Papers will be published in English, and authors are obliged to submit papers only in English, free of typing errors (please use Spell Checking). All titles of the papers listed in the list of references have to be in English, or translated in English with indication of the original language.

Editorial Board has right to make small lector corrections and to shorten text of the paper, which would not influence author's ideas and presentation. After computer lay-out of the paper, authors will obtain text as .pdf file for approval.

The manuscripts, not exceeding 16 pages, including figures and tables, should be submitted in electronic form only in MS Word (.doc file) or LaTeX format (.tex file), and sent to: okasn@rcub.bg.ac.rs and/or bakicv@vinca.rs

For each author full name (personal name and family or last name) and affiliation, have to be given. Family or last name has to be written in capital letters. In the case of more than one author, the name of corresponding author should be indicated, with full postal address, E-mail address, and affiliation.

Each paper has to be written according to following order: title, author(s), abstract, key words, body of the text with numerated sections and subsections including introduction and conclusions, acknowledgment (if necessary), nomenclature, and references. Pages must have page numbers.

Corresponding author will obtain 10 reprints of the paper.

Detailed **Instruction for preparing manuscripts** can be find at web site of the journal *Thermal Science*: <http://thermalscience.vinca.rs>

**The papers not prepared according to the instruction rules
will be returned to the authors for proper preparation.**

INVESTIGATION OF THE IONIZED GAS FLOW ADJACENT TO POROUS WALL IN THE CASE WHEN ELECTROCONDUCTIVITY IS A FUNCTION OF THE LONGITUDINAL VELOCITY GRADIENT

by

Slobodan R. SAVIĆ*, **Branko R. OBROVIĆ**,
Dušan R. GORDIĆ, and **Saša B. JOVANOVIĆ**

Faculty of Mechanical Engineering, University of Kragujevac,
Kragujevac, Serbia

Original scientific paper
UDC: 537.565:533.6.011
DOI: 10/2298/TSCI1001089S

This paper studies the laminar boundary layer on a body of an arbitrary shape when the ionized gas flow is planar and steady and the wall of the body within the fluid porous. The outer magnetic field is perpendicular to the fluid flow. The inner magnetic and outer electric fields are neglected. The ionized gas electroconductivity is assumed to be a function of the longitudinal velocity gradient. Using transformations, the governing boundary layer equations are brought to a general mathematical model. Based on the obtained numerical solutions in the tabular forms, the behaviour of important non-dimensional quantities and characteristics of the boundary layer is graphically presented. General conclusions about the influence of certain parameters on distribution of the physical quantities in the boundary layer are drawn.

Key words: *boundary layer, ionized gas electroconductivity, porous wall, porosity parameter*

Introduction

The dissociated gas flow have been studied by various investigators like Dorrance [1], Loitsianskii [2, 3], Krivstova [4, 5], Saljnikov [6], and Obrović [7]. They performed a detailed investigation of the dissociated gas flow in the boundary and achieved significant results. Boričić *et al.* [8-10] and Ivanović [11] studied MHD boundary layer on a non-porous and porous contour of the body within the fluid and tried to find the so-called auto-model solution. The ionized gas flow in the boundary layer adjacent to both non-porous body [12, 13] and porous body [14-17] of an arbitrary shape were also studied for different electroconductivity variation laws.

This paper studies a complex ionized gas (air) flow in the boundary layer adjacent to the porous wall in the case when the electroconductivity is a function of the longitudinal velocity gradient.

* Corresponding author; e-mail: ssavic@kg.ac.rs

Mathematical formulation

At high gas flow velocities (*e. g.* supersonic flight of an aircraft through the Earth's atmosphere), the temperature in the viscous boundary layer increases significantly. At high temperatures ionization of gas (air) occurs together with dissociation. Because of this thermochemical reaction the gas becomes electroconductive. Then the gas (air) consists of positively charged ions, electrons, and atoms of oxygen and nitrogen. If the ionized gas flows in the magnetic field of the power $B_m = B_{my} = B_m(x)$, an electric current is formed in the gas, which causes appearance of the Lorentz force and the Joule's heat. Due to these effects, new terms, not found in the equations for homogenous unionized gas, appear in the equations of the ionized gas boundary layer.

This paper investigates the ionized gas flow when the outer magnetic field is perpendicular to the wall of the body within the fluid. The magnetic Reynolds number is considered very small. The ionized gas of the same physical characteristics as the gas in the main flow, is injected, *i. e.*, ejected perpendicularly to the porous wall with the velocity $v_w(x)$. According to [1], the complete governing equation system with the corresponding boundary conditions takes the following form:

$$\frac{\partial}{\partial x}(\rho u) - \frac{\partial}{\partial y}(\rho v) = 0 \quad (1)$$

$$\rho u \frac{\partial u}{\partial x} - \rho v \frac{\partial u}{\partial y} - \frac{dp}{dx} - \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} - \sigma B_m^2 u \right) \quad (2)$$

$$\rho u \frac{\partial h}{\partial x} - \rho v \frac{\partial h}{\partial y} - u \frac{dp}{dx} - \mu \frac{\partial u}{\partial y}^2 - \frac{\partial}{\partial y} \left(\frac{\mu}{Pr} \frac{\partial h}{\partial y} - \sigma B_m^2 u^2 \right) \quad (3)$$

$$\begin{aligned} u &= 0, & v &= v_w(x), & h &= h_w & \text{for } y &= 0 \\ u &= u_e(x), & & & h &= h_e(x) & \text{for } y &= \infty \end{aligned} \quad (4)$$

The ionized gas electroconductivity σ is assumed to be a function of the longitudinal velocity gradient:

$$\sigma = \sigma_0 \frac{v_0}{u_e^2} \frac{\partial u}{\partial y}, \quad (\sigma_0, v_0 = \text{const.}) \quad (5)$$

Based on the boundary conditions for the velocity and the density at the outer edge of the boundary layer:

$$u(x, y) = u_e(x), \quad \frac{\partial u}{\partial y} = 0, \quad \rho = \rho_e \quad (6)$$

The pressure is eliminated from eqs. (2) and (3), and the following system is obtained:

$$\frac{\partial}{\partial x}(\rho u) - \frac{\partial}{\partial y}(\rho v) = 0 \quad (7)$$

$$\rho u \frac{\partial u}{\partial x} - \rho v \frac{\partial u}{\partial y} - \rho_e u_e \frac{du_e}{dx} - \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} - \sigma B_m^2 u \right) \quad (8)$$

$$\rho u \frac{\partial h}{\partial x} - \rho v \frac{\partial h}{\partial y} - u \rho_e u_e \frac{du_e}{dx} - \mu \frac{\partial u}{\partial y} - \frac{\partial}{\partial y} \left(\frac{\mu}{Pr} \frac{\partial h}{\partial y} \right) - \sigma B_m^2 u^2 \quad (9)$$

The boundary conditions remain unchanged.

Transformation of the variables

In order to apply the general similarity method, instead of physical coordinates x and y , new transformations [3]:

$$s(x) = \frac{1}{\rho_0 \mu_0} \int_0^x \rho_w \mu_w dx, \quad z(x, y) = \frac{1}{\rho_0} \int_0^y dy \quad (10)$$

and the stream function $\psi(s, z)$ are introduced:

$$u = \frac{\partial \psi}{\partial z}, \quad \tilde{v} = \frac{\rho_0 \mu_0}{\rho_w \mu_w} u \frac{\partial z}{\partial x} - v \frac{\partial}{\partial_0} = \frac{\partial \psi}{\partial s} \quad (11)$$

The quantities ρ_0 and μ_0 denote the known values of the density and the dynamic viscosity of the ionized gas at a concrete point.

By means of the transformations (10) and (11), the governing equation system together with the boundary conditions comes to:

$$\frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial^2 \psi}{\partial z} \frac{\partial^2 \psi}{\partial z^2} - \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} - v_0 \frac{\partial}{\partial z} \left(Q \frac{\partial^2 \psi}{\partial z^2} - \frac{\rho_0 \mu_0}{\rho_w \mu_w} \frac{\sigma_0 B_m^2}{\rho_0} \frac{v_0}{u_e^2} \frac{\partial^2 \psi}{\partial z^2} \frac{\partial \psi}{\partial z} \right) \quad (12)$$

$$\begin{aligned} & \frac{\partial \psi}{\partial z} \frac{\partial h}{\partial s} - \frac{\partial \psi}{\partial s} \frac{\partial h}{\partial z} - \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} \frac{\partial \psi}{\partial z} - v_0 Q \frac{\partial^2 \psi}{\partial z^2} - v_0 \frac{\partial}{\partial z} \left(\frac{Q}{Pr} \frac{\partial h}{\partial z} \right) \\ & + \frac{\rho_0 \mu_0}{\rho_w \mu_w} \frac{\sigma_0 B_m^2}{\rho_0} \frac{v_0}{u_e^2} \frac{\partial^2 \psi}{\partial z^2} \frac{\partial \psi}{\partial z}, \quad Q = \frac{\rho \mu}{\rho_w \mu_w} \end{aligned} \quad (13)$$

$$\frac{\partial \psi}{\partial z} = 0, \quad \frac{\partial \psi}{\partial s} = \frac{\mu_0}{\mu_w} v_w = \tilde{v}_w, \quad h = h_w \quad \text{for } z = 0 \quad (14)$$

$$\frac{\partial \psi}{\partial z} = u_e(s), \quad h = h_e(s) \quad \text{for } z = \infty$$

In order to solve the system (12)-(14), the momentum equation is derived:

$$\frac{dZ^{**}}{ds} = \frac{F_{mp}}{u_e} \quad (15)$$

While deriving the momentum eq. (15), the usual quantities in the boundary layer theory are introduced:

$$F_{mp} = 2[\zeta - (2 + H)f] + g - 2\Lambda \quad (16)$$

$$H = \frac{\Delta^*}{\Delta^{**}}; \quad \Delta^*(s) = \int_0^\infty \frac{\rho_e}{\rho} \frac{u}{u_e} dz, \quad \Delta^{**}(s) = \int_0^\infty \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) dz \quad (17)$$

$$f(s) = f_1(s) - u_e Z^{**}, \quad Z^{**} = \frac{\Delta^{**2}}{v_0} \quad (18)$$

$$g(s) = g_1(s) = u_e^{-1} N_\sigma \sqrt{v_0 Z^{**}} \quad (19)$$

$$N_\sigma = \frac{\rho_0 \mu_0}{\rho_w \mu_w} N, \quad N = \frac{\sigma_0 B_m^2}{\rho_0} \quad (20)$$

$$\tau_w(s) = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = \frac{\rho_w \mu_w}{\rho_0} \frac{u_e}{\Delta^{**}} \zeta, \quad \zeta(s) = \frac{\partial u}{\partial z} \Big|_{z=0} \Delta^{**} \quad (21)$$

In the momentum equation $\Lambda(s)$ is the porosity parameter, and it is:

$$\Lambda = \frac{\mu_0}{\mu_w} \frac{v_w \Delta^{**}}{v_0} = \frac{V_w \Delta^{**}}{v_0} \Lambda(s) \quad (22)$$

where V_w is a conditional transversal velocity at the inner edge of the boundary layer of the porous wall of the body within the fluid.

For the used electroconductivity variation law, in order to apply the general similarity method, the boundary conditions and the stream function on the wall of the body within the fluid should remain the same as with the non-porous wall. For that reason, a new stream function is introduced $\psi^*(s, z)$ by the relation:

$$\psi(s, z) = \psi_w(s) + \psi^*(s, z), \quad \psi^*(s, 0) = 0 \quad (23)$$

where $\psi(s, 0) = \psi_w(s)$ denotes the stream function of the flow adjacent to the wall of the body within the fluid.

Applying the relation (23), the system (12)-(14) is transformed into:

$$\frac{\partial \psi^*}{\partial z} \frac{\partial^2 \psi^*}{\partial s \partial z} = \frac{\partial \psi^*}{\partial s} \frac{\partial^2 \psi^*}{\partial z^2} - \frac{d\psi_w}{ds} \frac{\partial^2 \psi^*}{\partial z^2} - \frac{\rho_e}{\rho} u_e u_e v_0 \frac{\partial}{\partial z} Q \frac{\partial^2 \psi^*}{\partial z^2} - \frac{\sigma_0 B_m^2 v_0}{\rho_0} \frac{\rho_0 \mu_0}{u_e^2 \rho_w \mu_w} \frac{\partial^2 \psi^*}{\partial z^2} \frac{\partial \psi^*}{\partial z} \quad (24)$$

$$\frac{\partial \psi^*}{\partial z} \frac{\partial h}{\partial s} = \frac{\partial \psi^*}{\partial s} \frac{\partial h}{\partial z} - \frac{d\psi_w}{ds} \frac{\partial h}{\partial z} - \frac{\rho_e}{\rho} u_e u_e \frac{\partial \psi^*}{\partial z} v_0 Q \frac{\partial^2 \psi^*}{\partial z^2} - v_0 \frac{\partial}{\partial z} \frac{Q}{Pr} \frac{\partial h}{\partial z} - \frac{\sigma_0 B_m^2 v_0}{\rho_0} \frac{\rho_0 \mu_0}{u_e^2 \rho_w \mu_w} \frac{\partial^2 \psi^*}{\partial z^2} \frac{\partial \psi^*}{\partial z} \quad (25)$$

$$\psi^* = 0, \quad \frac{\partial \psi^*}{\partial z} = 0, \quad h = h_w \quad \text{for } z = 0 \quad (26)$$

$$\frac{\partial \psi^*}{\partial z} = u_e(s), \quad h = h_e(s) \quad \text{for } z = \infty$$

General mathematical model

In order to derive the generalized boundary layer equations it is necessary to introduce new transformations:

$$s = s, \quad \eta(s, z) = \frac{\sqrt{u_e^b}}{K(s)} z, \quad \psi^*(s, z) = \frac{u_e}{\sqrt{u_e^b}} K(s) \Phi[\eta, \kappa, (f_k), (g_k), (\Lambda_k)] \quad (27)$$

$$h(s, z) = h_1 \bar{h}[\eta, \kappa, (f_k), (g_k), (\Lambda_k)]$$

$$h_e = \frac{u_e^2}{2}, \quad h_1 = \text{const.}, \quad K(s) = \sqrt{a v_0 \int_0^s u_e^{b-1} ds}, \quad a, b = \text{const.} \quad (28)$$

where $\eta(s, z)$ is the newly introduced transversal variable, Φ – the newly introduced stream function, and \bar{h} – the non-dimensional enthalpy.

Some important quantities and characteristics of the boundary layer (16)-(21) can be written in the form of more suitable relations:

$$u = u_e \frac{\partial \Phi}{\partial \eta} \quad (29)$$

$$\Delta^{**}(s) = \frac{K(s)}{\sqrt{u_e^b}} B(s), \quad B(s) = \int_0^\infty \frac{\partial \Phi}{\partial \eta} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) d\eta \quad (30)$$

$$\frac{\Delta^*(s)}{\Delta^{**}(s)} = H = \frac{A(s)}{B(s)}, \quad A(s) = \int_0^\infty \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} d\eta \quad (31)$$

$$\zeta = B \frac{\partial^2 \Phi}{\partial \eta^2} \Big|_{\eta=0} \quad (32)$$

$$\frac{f}{B^2} = \frac{a u_e}{u_e^b} \int_0^s u_e^{b-1} ds \quad (33)$$

In the general similarity transformations (27), with the non-dimensional functions Φ and \bar{h} , a local parameter of the ionized gas compressibility $\kappa = f_0$, a set of the form parameters f_k [3], a set of magnetic parameters g_k , and a set of porosity parameters Λ_k [18] are introduced:

$$\kappa = f_0(s) = \frac{u_e^2}{2h_1} \quad (34)$$

$$f_k(s) = u_e^{k-1} u_e^{(k)} Z^{**k}, \quad (k = 1, 2, 3, \dots) \quad (35)$$

$$g_k(s) = u_e^{k-2} N_\sigma^{(k-1)} \sqrt{\frac{v_0}{Z^{**}}} Z^{**k} \quad (36)$$

$$\Lambda_k(s) = u_e^{k-1} \frac{V_w}{\sqrt{v_0}} \frac{Z^{**k}}{\sqrt{Z^{**}}} \quad (37)$$

They represent independent variables instead of the longitudinal variable s .

The local compressibility parameter $\kappa = f_0$ and the sets of parameters satisfy the following corresponding simple recurrent differential equations:

$$\frac{u_e}{u_e} f_1 \frac{d\kappa}{ds} = 2\kappa f_1 - \theta_0 \quad (38)$$

$$\frac{u_e}{u_e} f_1 \frac{df_k}{ds} [(k-1)f_1 - kF_{mp}] f_k - f_{k-1} \theta_k, \quad (k=1,2,3,\dots) \quad (39)$$

$$\frac{u_e}{u_e} f_1 \frac{dg_k}{ds} = (k-2)f_1 - k \frac{1}{2} F_{mp} g_k - g_{k-1} \gamma_k \quad (40)$$

$$\frac{u_e}{u_e} f_1 \frac{d\Lambda_k}{ds} = (k-1)f_1 - \frac{2k-1}{2} F_{mp} \Lambda_k - \Lambda_{k-1} \chi_k \quad (41)$$

Applying the similarity transformations (27) and (34)-(37) the system (24)-(26) can be written as:

$$\begin{aligned} \frac{\partial}{\partial \eta} Q \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{aB^2}{2B^2} (2-b)f_1 \Phi \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{f_1}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} - \frac{g_1}{B^2} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} \\ - \frac{\Lambda_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{1}{B^2} \sum_{k=0}^{\infty} \theta_k \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial^2 \Phi}{\partial \eta^2} - \sum_{k=1}^{\infty} \gamma_k \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial g_k} - \frac{\partial \Phi}{\partial g_k} \frac{\partial^2 \Phi}{\partial \eta^2} \\ - \sum_{k=1}^{\infty} \chi_k \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial^2 \Phi}{\partial \eta^2} \end{aligned} \quad (42)$$

$$\begin{aligned} \frac{\partial}{\partial \eta} \frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} - \frac{aB^2}{2B^2} (2-b)f_1 \Phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f_1}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} - 2\kappa Q \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{2\kappa g_1}{B} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} \\ - \frac{\Lambda_1}{B} \frac{\partial \bar{h}}{\partial \eta} - \frac{1}{B^2} \sum_{k=0}^{\infty} \theta_k \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f_k} - \frac{\partial \Phi}{\partial f_k} \frac{\partial \bar{h}}{\partial \eta} \\ - \sum_{k=1}^{\infty} \gamma_k \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial g_k} - \frac{\partial \Phi}{\partial g_k} \frac{\partial \bar{h}}{\partial \eta} - \sum_{k=1}^{\infty} \chi_k \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial \Lambda_k} - \frac{\partial \Phi}{\partial \Lambda_k} \frac{\partial \bar{h}}{\partial \eta} \end{aligned} \quad (43)$$

The transformed boundary conditions are:

$$\begin{aligned} \Phi - \frac{\partial \Phi}{\partial \eta} = 0, \quad \bar{h} = \bar{h}_w \text{ const. for } \eta = 0 \\ \frac{\partial \Phi}{\partial \eta} = 1, \quad \bar{h} = \bar{h}_e, \quad 1 - \kappa \text{ for } \eta = \infty \end{aligned} \quad (44)$$

Neither eqs. (42) and (43) nor the boundary conditions (44) contain the outer velocity of the boundary layer. Therefore, this equation system is generalized and it represents a general mathematical model of the ionized gas flow adjacent to the porous wall of the body within the fluid for the assumed electroconductivity variation law (5).

Numerical solution

When the generalized equation system (42)-(43) with the boundary conditions (44) is numerically solved, a finite number of parameters is adopted and the solution is obtained in n -parametric approximation. Due to many difficulties in solution of this equation system, it can be solved only with a relatively small number of parameters. If it is assumed that:

$$\kappa f_0 = 0, f_1 = f = 0, g_1 = g = 0, \Lambda_1 = \Lambda = 0 \quad (45)$$

$$f_2 = f_3 = \dots = 0, g_2 = g_3 = \dots = 0, \Lambda_2 = \Lambda_3 = \dots = 0 \quad (46)$$

the obtained equation system is significantly simplified. Furthermore, when the general similarity method is applied, the so-called localization is performed. If we neglect derivatives per the compressibility, magnetic, and porosity parameters ($\kappa = 0, \Lambda = 0$), the equation system (42)-(43) is significantly simplified, and in the four-parametric three times localized approximation, it has the following form:

$$\frac{\partial}{\partial \eta} Q \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{aB^2 (2-b)f}{2B^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{f}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} - \frac{g}{B^2} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} - \frac{\Lambda}{B} \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{F_{mp} f}{B^2} \frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial \eta \partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial^2 \Phi}{\partial \eta^2} \quad (47)$$

$$\frac{\partial}{\partial \eta} \frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} - \frac{aB^2 (2-b)f}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f}{B^2} \frac{\rho_e}{\rho} \frac{\partial \Phi}{\partial \eta} - 2\kappa Q \frac{\partial^2 \Phi}{\partial \eta^2} - \frac{2\kappa g}{B} \frac{\partial^2 \Phi}{\partial \eta^2} \frac{\partial \Phi}{\partial \eta} - \frac{\Lambda}{B} \frac{\partial \bar{h}}{\partial \eta} - \frac{F_{mp} f}{B^2} \frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{h}}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \bar{h}}{\partial \eta} \quad (48)$$

The boundary conditions (44) remain unchanged.

In the equations of the system (47)-(48) the subscript 1 is left out in some parameters. Each of the equations contains a term that characterizes the porous wall of the body within the fluid.

For the numerical integration of the obtained system of differential partial equations of the third order, it is necessary to decrease the order of the differential equations. Using [6]:

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi = \varphi(\eta, \kappa, f, g, \Lambda) \quad (49)$$

the order of the differential equations is decreased, so the system together with the boundary conditions comes to:

$$\frac{\partial}{\partial \eta} Q \frac{\partial \varphi}{\partial \eta} - \frac{aB^2 (2-b)f}{2B^2} \Phi \frac{\partial \varphi}{\partial \eta} - \frac{f}{B^2} \frac{\rho_e}{\rho} \varphi^2 - \frac{g}{B} \frac{\partial \varphi}{\partial \eta} \varphi - \frac{\Lambda}{B} \frac{\partial \varphi}{\partial \eta} - \frac{F_{mp} f}{B^2} \varphi \frac{\partial \varphi}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \varphi}{\partial \eta} \quad (50)$$

$$\frac{\partial}{\partial \eta} \frac{Q}{Pr} \frac{\partial \bar{h}}{\partial \eta} - \frac{aB^2 (2-b)f}{2B^2} \Phi \frac{\partial \bar{h}}{\partial \eta} - \frac{2\kappa f}{B^2} \frac{\rho_e}{\rho} \varphi - 2\kappa Q \frac{\partial \varphi}{\partial \eta} - \frac{2\kappa g}{B} \frac{\partial \varphi}{\partial \eta} \varphi^2 - \frac{\Lambda}{B} \frac{\partial \bar{h}}{\partial \eta} - \frac{F_{mp} f}{B^2} \varphi \frac{\partial \bar{h}}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \bar{h}}{\partial \eta} \quad (51)$$

$$\begin{aligned} \Phi = \varphi = 0, \bar{h} = \bar{h}_w = \text{const. for } \eta = 0 \\ \varphi = 1, \bar{h} = \bar{h}_e = 1 - \kappa \text{ for } \eta = \infty \end{aligned} \quad (52)$$

In order to solve the obtained system (50)-(52), it is necessary to determine the analytic forms of distribution of certain physical quantities that are themselves part of the equations. For the non-dimensional function Q [15] and the density ratio ρ_e/ρ [4], the following expressions are adopted:

$$Q = Q(\bar{h}) \sqrt[3]{\frac{\bar{h}_w}{h}}, \quad \frac{\rho_e}{\rho} = \frac{\bar{h}}{1 - \kappa} \quad (53)$$

A concrete numerical solution of the obtained system of non-linear and conjugated differential partial equations (50)-(52) is performed using finite differences method, *i. e.*, "passage method" or TDA method. Based on the scheme of the plane integration grid [6], the system (50)-(52) is brought to the following system of linear algebraic equations:

$$a_{M,K-1}^i \varphi_{M-1,K-1}^j - 2b_{M,K-1}^i \varphi_{M,K-1}^j + c_{M,K-1}^i \varphi_{M+1,K-1}^j - g_{M,K-1}^i \quad (54)$$

$$a_{M,K-1}^j \bar{h}_{M-1,K-1}^j - 2b_{M,K-1}^j \bar{h}_{M,K-1}^j + c_{M,K-1}^j \bar{h}_{M+1,K-1}^j - g_{M,K-1}^j \quad (55)$$

$$M = 2, 3, \dots, N-1; \quad K = 0, 1, 2, \dots, \quad i, j = 0, 1, 2, \dots$$

$$\begin{aligned} \Phi_{1,K-1}^i &= \varphi_{1,K-1}^i = 0, \quad \bar{h}_{1,K-1}^j = \bar{h}_w \text{ const. for } M=1 \\ \varphi_{N,K+1}^i &= 1, \quad \bar{h}_{N,K+1}^j = \frac{1}{1-\kappa} \text{ for } M=N \end{aligned} \quad (56)$$

The coefficients $a_{M,K-1}^i$, $b_{M,K-1}^i$, $c_{M,K-1}^i$, and $g_{M,K-1}^i$ of the dynamic equation are determined with the expressions:

$$\begin{aligned} a_{M,K-1}^i &= Q_{M,K-1}^{j-1} - \frac{1}{4} (Q_{M,K-1}^{j-1} - Q_{M,K-1}^{j+1}) - \frac{\Delta\eta}{2(B_{K-1}^{i-1})^2} - a(B_{K-1}^{i-1})^2 - (2-b)f_{K-1} - \frac{\Phi_{M,K-1}^{i-1}}{2} \\ &F_{mp,K-1}^{i-1} f_{K-1} - \frac{\Phi_{M,K-1}^{i-1} - \Phi_{M,K-1}}{\Delta f} - \frac{\Delta\eta}{2} \frac{g}{B_{K-1}^{i-1}} \varphi_{M,K-1}^{i-1} - \frac{\Delta\eta}{2} \frac{\Lambda}{B_{K-1}^{i-1}} \end{aligned} \quad (57)$$

$$b_{M,K-1}^i = Q_{M,K-1}^{j-1} - \frac{(\Delta\eta)^2}{2(B_{K-1}^{i-1})^2} f_{K-1} \varphi_{M,K-1}^{i-1} - \frac{F_{mp,K-1}^{i-1}}{\Delta f} \quad (58)$$

$$\begin{aligned} c_{M,K-1}^i &= Q_{M,K-1}^{j+1} - \frac{1}{4} (Q_{M,K-1}^{j-1} - Q_{M,K-1}^{j+1}) - \frac{\Delta\eta}{2(B_{K-1}^{i-1})^2} - a(B_{K-1}^{i-1})^2 - (2-b)f_{K-1} - \frac{\Phi_{M,K-1}^{i-1}}{2} \\ &F_{mp,K-1}^{i-1} f_{K-1} - \frac{\Phi_{M,K-1}^{i-1} - \Phi_{M,K-1}}{\Delta f} - \frac{\Delta\eta}{2} \frac{g}{B_{K-1}^{i-1}} \varphi_{M,K-1}^{i-1} - \frac{\Delta\eta}{2} \frac{\Lambda}{B_{K-1}^{i-1}} \end{aligned} \quad (59)$$

$$g_{M,K-1}^i = \frac{(\Delta\eta)^2}{(B_{K-1}^{i-1})} f_{K-1} \frac{\bar{h}_{M,K-1}^j}{1-\kappa} - F_{mp,K-1}^{i-1} f_{K-1} \varphi_{M,K-1}^{i-1} - \frac{\Phi_{M,K-1}}{\Delta f} \quad (60)$$

For the thermodynamic equation, these coefficients are:

$$a_{M,K-1}^j = \frac{Q_{M,K-1}^{j-1}}{\text{Pr}} - \frac{1}{4\text{Pr}} (Q_{M-1,K-1}^{j-1} - Q_{M,1,K-1}^{j-1}) - \frac{\Delta\eta}{2(B_{K-1}^{i-1})^2} [a(B_{K-1}^{i-1})^2 - (2-b)f_{K-1}] \frac{\Phi_{M,K-1}^{i-1} F_{mp,K-1}^{i-1} f_{K-1}}{2} - \frac{\Phi_{M,K-1}^{i-1} \Phi_{M,K}}{\Delta f} \frac{\Delta\eta}{2} \frac{\Lambda}{B_{K-1}^{i-1}} \quad (61)$$

$$b_{M,K-1}^j = \frac{Q_{M,K-1}^{j-1}}{\text{Pr}} - \frac{(\Delta\eta)^2}{2(B_{K-1}^{i-1})^2} f_{K-1} \varphi_{M,K-1}^{i-1} - \frac{2\kappa}{1-\kappa} \frac{F_{mp,K-1}^{i-1}}{\Delta f} \quad (62)$$

$$c_{M,K-1}^j = \frac{Q_{M,K-1}^{j-1}}{\text{Pr}} - \frac{1}{4\text{Pr}} (Q_{M-1,K-1}^{j-1} - Q_{M,1,K-1}^{j-1}) - \frac{\Delta\eta}{2(B_{K-1}^{i-1})^2} [a(B_{K-1}^{i-1})^2 - (2-b)f_{K-1}] \frac{\Phi_{M,K-1}^{i-1} F_{mp,K-1}^{i-1} f_{K-1}}{2} - \frac{\Phi_{M,K-1}^{i-1} \Phi_{M,K}}{\Delta f} \frac{\Delta\eta}{2} \frac{\Lambda}{B_{K-1}^{i-1}} \quad (63)$$

$$g_{M,K-1}^j = \frac{(\Delta\eta)^2}{(B_{K-1}^{i-1})^2} F_{mp,K-1}^{i-1} f_{K-1} \varphi_{M,K-1}^{i-1} - \frac{\bar{h}_{M,K}}{\Delta f} - \frac{\kappa}{2} Q_{M,K-1}^{j-1} (\varphi_{M-1,K-1}^{i-1} - \varphi_{M,1,K-1}^{i-1})^2 - \frac{\Delta\eta}{B_{K-1}^{i-1}} \kappa g (\varphi_{M-1,K-1}^{i-1} - \varphi_{M,1,K-1}^{i-1}) (\varphi_{M,K-1}^{i-1})^2 \quad (64)$$

From the algebraic eqs. (54)-(56), the following formulae are obtained :

$$\varphi_{N,K-1}^{i-1} = 1 \quad (65)$$

$$\varphi_{M,K-1}^i = K_{M,K-1}^i - L_{M,K-1}^i \varphi_{M-1,K-1}^i \quad (66)$$

$$\varphi_{1,K-1}^i = 0 \quad (67)$$

$$\bar{h}_{N,K-1}^j = 1 - \kappa \quad (68)$$

$$\bar{h}_{M,K-1}^j = K_{M,K-1}^j - L_{M,K-1}^j \bar{h}_{M-1,K-1}^j \quad (69)$$

$$\bar{h}_{1,K-1}^j = \bar{h}_w = \text{const.} \quad (70)$$

$M = N - 1, N - 2, \dots, 3, 2; \quad i, j = 1, 2, 3, \dots$

and they are used to calculate the values of the functions φ and \bar{h} at discrete points in the direction of decrease of the subscript M .

In the formulae (65)-(70), the passage coefficients for the dynamic equations are:

$$K_{M,K-1}^i = \frac{a_{M,K-1}^i K_{M-1,K-1}^i - g_{M,K-1}^i}{2b_{M,K-1}^i - a_{M,K-1}^i L_{M-1,K-1}^i}, \quad K_{1,K-1}^i = \varphi_{1,K-1}^i = 0 \quad (71)$$

$$L_{M,K-1}^i = \frac{c_{M,K-1}^i}{2b_{M,K-1}^i - a_{M,K-1}^i L_{M-1,K-1}^i}, \quad L_{1,K-1}^i = 0 \quad (72)$$

These coefficients for the thermodynamic equation have the same form but they are essentially different:

$$K_{M,K-1}^j = \frac{a_{M,K-1}^j K_{M-1,K-1}^j g_{M,K-1}^j}{2b_{M,K-1}^j a_{M,K-1}^j L_{M-1,K-1}^j}, \quad K_{1,K-1}^j = \bar{h}_{1,K-1}^j = \bar{h}_w = \text{const.} \quad (73)$$

$$L_{M,K-1}^j = \frac{c_{M,K-1}^j}{2b_{M,K-1}^j a_{M,K-1}^j L_{M-1,K-1}^j}, \quad L_{1,K-1}^j = 0 \quad (74)$$

$M = 2, 3, \dots, N-2, N-1$

Based on the recurrent formulae (71)-(74), the passage coefficients in the direction of the increase of the subscript M are calculated. After all the discrete points of the calculating layer have been gone through twice, the solutions of the functions φ and \bar{h} that correspond to that layer are calculated. The procedure is then repeated for all the calculating layers of the plane integration grid until the integration is performed in the whole range of the possible change of the parameter of the form f . Based on [13], the number of nodes is determined for each calculating layer as $N = 401$.

Prandtl number depends little on the temperature, therefore in this paper its value is considered to be constant and for air it is $Pr = 0.712$. According to [6], the optimal values for the constants a and b are: $a = 0.4408$, $b = 5.7140$.

Results

For the numerical solution of the equation system (50)-(52), a program in FORTRAN program language has been written. As the first derivative is neglected due to localization per the compressibility, porosity, and magnetic parameters, the program is designed to enable the solution of the equations for in advance given values of these now simple parameters. Numerical solutions are obtained in the output database in the tabular form.

The following results have been obtained.

Regardless of the fact whether the ionized gas is injected into the main flow or ejected from it, at different cross-sections of the boundary layer, the non-dimensional velocity u/u_e very quickly converges towards unity (fig. 1).

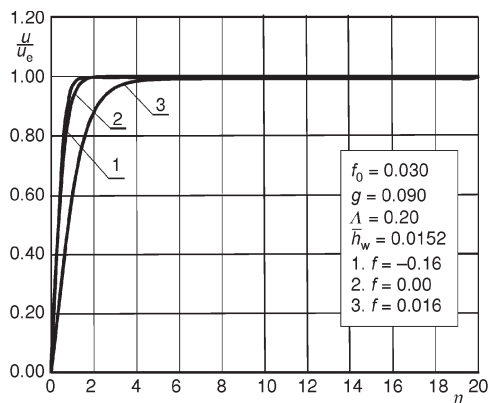


Figure 1. Diagram of the non-dimensional velocity u/u_e

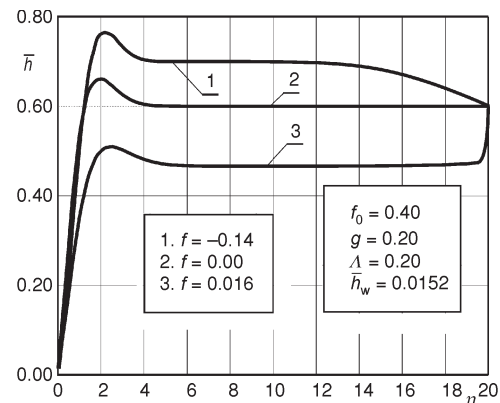


Figure 2. Distribution of the non-dimensional enthalpy \bar{h}

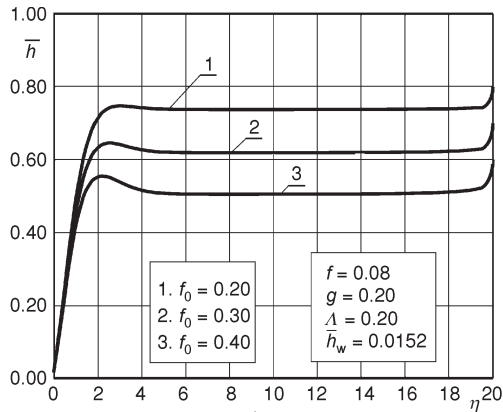


Figure 3. Distribution of the non-dimensional enthalpy for different values of the parameter f_0

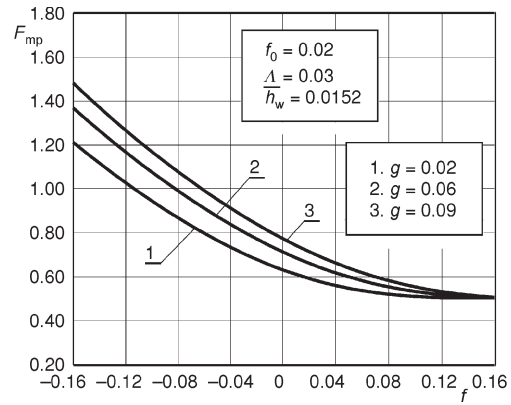


Figure 4. Distribution of the characteristic function F_{mp}

The compressibility parameter $\kappa = f_0$ has little influence on the corresponding distributions of the non-dimensional velocity.

In the presented (figs. 2 and 3) and other diagrams for distribution of the non-dimensional enthalpy we notice a great change of the enthalpy near the wall of the body within the fluid and near the outer edge of the boundary layer.

The change of the porosity parameter has a great influence on the distribution of the non-dimensional enthalpy \bar{h} in the ionized gas boundary layer (fig. 3).

The magnetic field has a great influence on the characteristic of the boundary layer F_{mp} (fig. 4) and the non-dimensional friction function ζ . By increasing the values of the magnetic parameter, the separation of the boundary layer is postponed (fig. 5).

Based on the diagrams that are not presented here, it can be concluded that variation of the porosity parameter has little influence on the profiles of the non-dimensional velocities u/u_e .

The porosity parameter Λ has a great influence on the non-dimensional friction function ζ (fig. 6). Consequently, it also has a great influence on the boundary layer separation point.

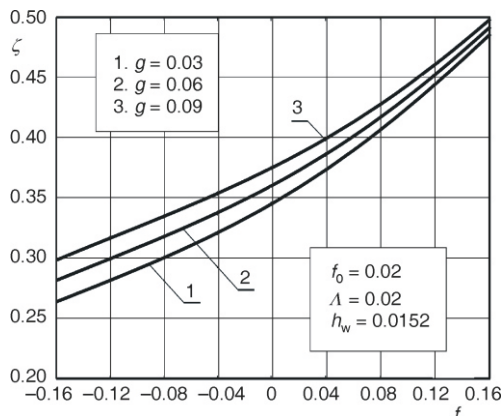


Figure 5. Distribution of the non-dimensional friction function $\zeta(g)$

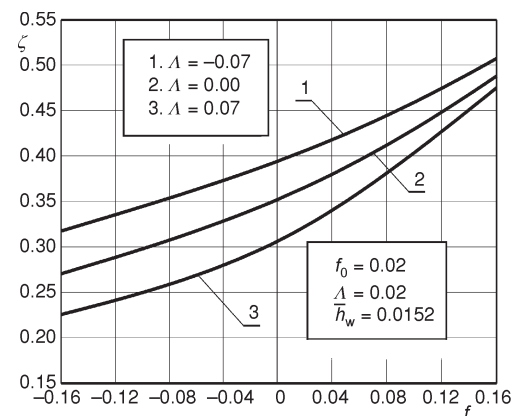


Figure 6. Distribution of the non-dimensional friction function $\zeta(\Lambda)$

It is noted that the injection of air postpones the separation of the ionized gas boundary layer because the separation point moves down the flow.

This parameter has a significant influence on the characteristic function of the boundary layer on the porous wall F_{mp} .

Conclusions

This paper studies the ionized gas planar steady flow in the boundary layer adjacent to the porous wall. The ionized gas of the same characteristics as the gas in the main current is injected *i. e.*, ejected perpendicularly to the wall. The outer magnetic field is perpendicular to the contour of the body. The gas electroconductivity is assumed a function of the longitudinal velocity gradient.

The aim of the investigation is to apply the general similarity method to the studied problem and solve the obtained equations. The governing equation system is transformed, brought to a general form, and then numerically solved by application of the finite differences method. However, the numerical solution is fraught with difficulties, mainly of mathematical nature, although there are some difficulties related to thermochemical and physical processes of the gas flow.

Complex fluid flow problems can be successfully solved using general similarity method. Distributions of the solutions of the ionized gas boundary layer equations for the used electroconductivity variation law are shown to be same as with other similar compressible fluid flow problems. Some new facts about the influence of the magnetic field and the porosity on the boundary layer separation have also been discovered. Important quality results here obtained enable an insight in the distribution of physical and characteristic quantities at different cross-sections of the boundary layer.

Acknowledgment

The research was supported by the Ministry of Science and Environmental Protection of the Republic of Serbia Grant ON144022.

Nomenclature

A, B	– boundary layer characteristics, [–]	h_w	– enthalpy at the wall of the body within the fluid, [Jkg ⁻¹]
B_m	– induction of outer magnetic field [= $B_m(x)$] [Vsm ⁻²]	h_1	– enthalpy at the front stagnation point of the body within the fluid, [Jkg ⁻¹]
a, b	– constants, [–]	i, j	– iteration number, [–]
c_p	– specific heat of ionized gas at constant pressure, [Jkg ⁻¹ K ⁻¹]	M	– discrete point, [–]
F_{mp}	– characteristic boundary layer function, [–]	Pr	– Prandtl number (= $\mu c_p / \lambda$), [–]
f_1	– first form parameter (= f), [–]	p	– pressure, [Pa]
f_k	– set of form parameters, [–]	Q	– non-dimensional function, [–]
g_1	– first magnetic parameter (= g), [–]	s	– new longitudinal variable, [m]
g_k	– set of magnetic parameters, [–]	u	– longitudinal projection of velocity in the boundary layer, [ms ⁻¹]
H	– boundary layer characteristic, [–]	u_e	– velocity at the boundary layer outer edge, [ms ⁻¹]
h	– enthalpy, [Jkg ⁻¹]	V_w	– conditional transversal velocity, [ms ⁻¹]
\bar{h}	– non-dimensional enthalpy, [–]		
h_e	– enthalpy at the outer edge of the boundary layer, [Jkg ⁻¹]		

v	– transversal projection of velocity in the boundary layer, [ms ⁻¹]	μ_0	– known values of dynamic viscosity of the ionized gas, [Pa·s]
v_w	– velocity of injection (or ejection) of the fluid, [ms ⁻¹]	μ_w	– given distributions of dynamic viscosity at the wall of the body within the fluid, [Pa·s]
x, y	– longitudinal and transversal coordinate, [m]	ν_0	– kinematic viscosity at a concrete point of the boundary layer, [m ² s ⁻¹]
Z^{**}	– function, [s]	ρ	– density of ionized gas, [kgm ⁻³]
z	– new transversal variable, [m]	ρ_e	– ionized gas density at the outer edge of the boundary layer, [kgm ⁻³]
Greek symbols			
Δ^*	– conditional displacement thicknesses, [m]	ρ_w	– given distributions of density at the wall of the body within the fluid, [kgm ⁻³]
Δ^{**}	– conditional momentum loss thickness, [m]	ρ_0	– known values of density of the ionized gas, [kgm ⁻³]
ζ	– non-dimensional friction function, [-]	σ	– electroconductivity, [Nm ³ V ⁻² s ⁻¹]
η	– non-dimensional transversal coordinate, [-]	τ_w	– shear stress at the wall of the body within the fluid, [Nm ⁻²]
κ	– local compressibility parameter, (= f_0) [-]	Φ	– non-dimensional stream function, [-]
Λ_1	– first porosity parameter (= Λ), [-]	ψ	– stream function, [m ² s ⁻¹]
Λ_k	– set of porosity parameters, [-]	ψ^*	– new stream function, [m ² s ⁻¹]
λ	– thermal conductivity coefficient, [Wm ⁻¹ K ⁻¹]		
μ	– dynamic viscosity, [Pa·s]		

References

- [1] Dorrance, W. H., Viscous Hypersonic Flow, Theory of Reacting and Hypersonic Boundary Layers (in Russian), Mir, Moscow, 1966
- [2] Loitsianskii, L. G., Laminar Boundary Layer (in Russian), FML, Moscow, 1962
- [3] Loitsianskii, L. G., Liquid and Gas Mechanics (in Russian), Nauka, Moscow, 1978
- [4] Krivtsova, N. V., Laminar Boundary Layer with the Equilibrium Dissociated Gas (in Russian), Gidrogazodinamika, Trudi LPI, 265 (1966), pp. 35-45
- [5] Krivtsova, N. V., Parameter Method of Solving the Laminar Boundary Layer Equations with Axial Pressure Gradient in the Conditions of Equilibrium Dissociation of the Gas (in Russian), Engineering-Physical Journal, 10 (1966), 2, pp. 143-153
- [6] Saljnikov, V., Dallmann, U., Generalized Similarity Solutions for Three Dimensional, Laminar, Steady Compressible Boundary Layer Flows on Swept, Profiled Cylinders (in German), Institute for Theoretical Fluid Mechanics, DLR-FB 89-34, Göttingen, Germany, 1989
- [7] Obrović, B., Boundary Layer of Dissociated Gas (in Serbian), Monograph, Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia, 1994
- [8] Boričić, Z., Nikodijević, D., Obrović, B., Unsteady Flow of the Liquid Whose Electroconductivity is a Function of the Longitudinal Velocity Gradient in MHD Boundary Layer on a Body (in Serbian), Proceedings, 20th Yugoslav Congress of Theoretical and Applied Mechanics, Kragujevac, Serbia, 1993, pp. 136-139
- [9] Boričić, Z., Nikodijević, D., Milenković, D., Unsteady MHD Boundary Layer on a Porous Surface, Facta Universitatis, Series: Mechanics, Automatic Control and Robotics, 1 (1995), 5, pp. 631-643
- [10] Saljnikov, V., Boričić, Z., Nikodijević, D., Parametric Method in Unsteady MHD Boundary Layer Theory of Fluid with Variable Electroconductivity, Facta Universitatis, Series: Mechanics, Automatic Control and Robotics, 2 (1997), 7/2, pp. 331-340
- [11] Ivanović, D., Unsteady Incompressible MHD Boundary Layer on Porous Aerofoil in High Accelerating Fluid Flow, Theoret. Appl. Mech., 27 (2002), 1, pp. 87-102
- [12] Obrović, B., Boričić, Z., Savić, S., Boundary Layer of Ionized Gas in the Case of Changeable Electroconductivity, Facta Universitatis, Series: Mechanics, Automatic Control and Robotics, 2 (1999), 9, pp. 953-963
- [13] Saljnikov, V., Obrović, B., Savić, S., Ionized Gas Flow in the Boundary Layer for Different Forms of the Electroconductivity Variation Flow, Theoret. Appl. Mech., 26 (2001), 1, pp. 15-31

- [14] Obrović, B., Savić, S., Ionized Gas Boundary Layer on a Porous Wall of the Body within the Electroconductive Fluid, *Theoret. Appl. Mech.*, 31 (2004), 1, pp. 47-71
- [15] Savić, S., Solution of the Problem of the Ionized Gas Flow in the Boundary Layer in Case of a Nonporous and a Porous Contour of the Body within the Fluid (in Serbian), Ph. D. thesis, Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, Serbia, 2006
- [16] Savić, S., Obrović, B., The Influence of Variation of Electroconductivity on Ionized Gas Flow in the Boundary Layer along a Porous Wall, *Theoret. Appl. Mech.*, 33 (2006), 2, pp. 149-179
- [17] Obrović, B., Savić, S., Ionized Gas Boundary Layer on a Porous Wall of the Body whose Electroconductivity is a Function of the Velocity Ratio, *Facta Universitatis, Series: Mechanics, Automatic Control and Robotics*, 6 (2007), 1, pp. 145-160
- [18] Obrović, B., Savić, S., About Porosity Parameters with the Application of the General Similarity Method to the Case of a Dissociated Gas Flow in the Boundary Layer, *Kragujevac J. Math.*, 24 (2002), 1, pp. 207-214